Ninth Itinerant Workshop in PDEs
Institut de Mathématiques de Bordeaux
January 8-10, 2018

Abstracts

Asymptotic dynamics of non-homogeneous fluids in fast rotation

Francesco Fanelli

In this talk we consider a class of singular perturbation problems for systems of PDEs related to the dynamics of geophysical fluids. We are interested in effects due to both density variations and Earth rotation, and to their interplay. We specialize on the 2-D non-homogeneous incompressible Navier-Stokes equations with Coriolis force: our goal is to characterize the asymptotic dynamics of weak solutions to this model, in the limit when the rotation becomes faster and faster.

We present two kinds of results, deeply different from each other from a qualitative viewpoint. If the initial density is a small perturbation of a constant state, we prove that the limit dynamics is essentially described by a homogeneous Navier-Stokes system with an additional forcing term, which can be seen as a remainder of density variations. If, instead, the initial density is a small perturbation of a truly variable reference state, we show that the final equations become linear, and moreover one can identify only a mean motion, described in terms of the limit vorticity and the limit density fluctuation function; this issue can be interpreted as a sort of turbulent behaviour of the limit flow.

This talk is based on a joint work with Isabelle Gallagher.

Unique Continuation for the Schrödinger equation on homogeneous trees

Aingeru Fernández-Bertolin

In this talk, we will see that if a solution of the time-dependent Schrödinger equation

$$\partial_t u = \Delta u + V u$$

on a homogeneous tree (with Δ the graph Laplacian) decays fast enough at two distinct times then the solution is trivial. This can be understood as a dynamic version of the Hardy Uncertainty Principle on homogeneous trees, a classical result in Harmonic Analysis proved via complex analysis.

We will show how to use complex analysis and spectral decompositions of Schrödinger operators to extend the classical results to homogeneous trees.

Since $\mathbb{Z}$ can be understood as a homogeneous tree of degree 1, we will focus on this setting to show also a proof based on real calculus, based on the results of Escauriaza, Kenig, Ponce and Vega in $\mathbb{R}^d$, and then we will extend the proof to homogeneous trees of degree $q \geq 2$.

This is joint work with Ph. Jaming (Bordeaux) and L. Vega (UPV/EHU and BCAM)

Asymptotic behaviour for fractional diffusion-convection equations

Liviu Ignat

In this talk we analyze the long time behaviour of the solutions of the equation

$$u_t(t,x) + (-\Delta)^{\alpha/2} u(t,x) + (f(u))_x = 0, \quad t > 0, \quad x \in \mathbb{R},$$

where $\alpha \in (0,2)$ and $f(s) = |s|^{q-1}s/q$ with $q > 1$. We present some previous results on the asymptotic expansion of the solutions when the time goes to infinity. We prove that in the one-dimensional case, for $1 < q < \alpha < 2$ the asymptotic behaviour is given by the entropy solution of the conservation law $u_t(t,x) + (f(u))_x = 0$, $u(0) = M\delta_0$ where M is the mass of the initial data. The proof relays on tricky inequalities to guarantee an Oleinik type inequality $(u^{q_1})_x \leq 1/t$.

An invitation to (some) nonlocal Hamiltonian PDEs

Enno Lenzmann

Hamiltonian PDEs with nonlocal dispersion (e.g. given by the fractional Laplacian) arise in a broad variety of physical models, where long-range interactions typically play a dominant role. In this mini-course, we will give an introduction to various techniques that allow us to study uniqueness and nondegeneracy for solitary waves for nonlinear equation with the fractional Laplacian. In particular, we present techniques to study nonlinear scalar equations (e.g. ground states for fractional nonlinear Schrödinger equations) as well as vectorial problems (e.g. half-harmonic maps).

Essential parts of this mini-course is based on joint work with R. Frank, L. Silvestre and A. Schikorra.

Topological solitons in chiral magnetism

Christof Melcher

Chiral skyrmions are topological solitons occurring in magnets without in- version symmetry. In this talk I shall explain the analytical structure and variational consequences of chiral interactions responsible for the occurrence of new magnetic phases in the spirit of Ginzburg-Landau, and for the stabilization and dynamics of magnetic skyrmions.

On the stability of columnar vortices

Didier Smets

The lecture will first review some classical works by Kelvin, Rayleigh and Helmholtz regarding the stability of columnar vortices in inviscid incompressible fluids, with respect to perturbations that are either 2D or 3D axisymmetric. The focus will then be put on arbitrary 3D perturbations, and the presentation of recent joint work with Thierry Gallay (Grenoble) that establishes 3D linear stability for a large class of columnar vortices, including the well-known Lamb-Oseen vortex. Time depending, some comparisons with the case of viscous or quantum fluids will be discussed too.

The structure of the free boundary for the thin obstacle problem

Emanuele Spadaro

In this talk I present some recent results on the global structure of the free boundary for a class of lower dimensional obstacle problems (including the classical scalar Signorini problem), in particular showing the local finiteness of the free boundary and its rectifiability. Connections with the fractional obstacle problem will also be discussed. This is a joint work with M. Focardi (Univ. of Firenze).